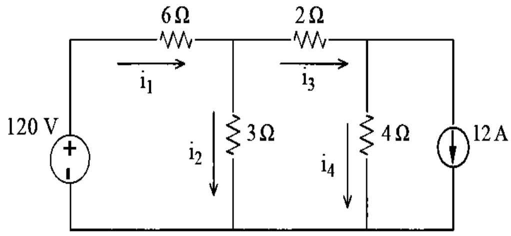
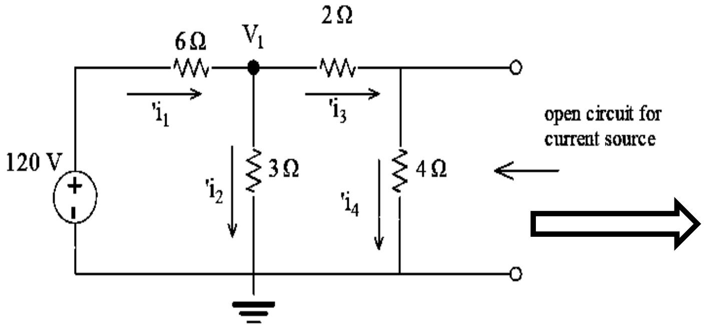
Chapter 3 solved problem

1. Use superposition to find i1, i2, i3, i4?



Solution

- If independent voltage source is activated 120 V on



$$\frac{120-V_1}{6}-\frac{V_1}{3}-\frac{V_1}{2+4}=0$$

$$20-V_1\left(\frac{1}{6}+\frac{1}{3}+\frac{1}{6}\right)=0$$

$$\Rightarrow \quad V_1=30\ \text{V}$$

•Using KCL at V1 (nodal analysis)

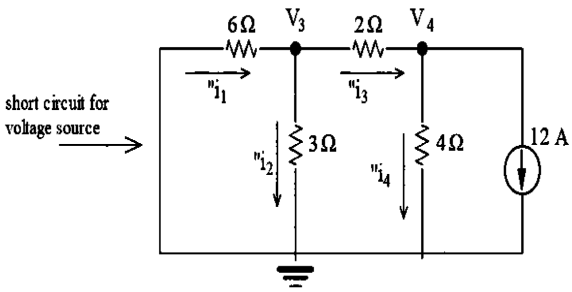
$$'i_1-'i_2-'i_3=0$$

$$'i_1=\frac{120-V_1}{6}=\frac{90}{6}=15\ \text{A}$$

$$'i_2=\frac{V_1}{3}=\frac{30}{3}=10\ \text{A}$$

$$'i_3=i_4=\frac{V_1}{6}=5\ \text{A}$$

- If independent current source is activated



KCL at V_3:

$$"i_1-"i_2-"i_3=0$$

$$\frac{-V_3}{6}-\frac{V_3}{3}-\frac{V_3-V_4}{2}=0$$

$$-V_3-2\,V_3-3\,(V_3-V)=0$$

$$-6\,V_3+3V_4=0 \qquad ......(1)$$

KCL at V4:

$$"i_3-"i_4-12=0$$

$$\frac{V_3-V_4}{2}-\frac{V_4}{4}-12=0$$

$$2\,V_3-2\,V_4-V_4=48$$

$$2\,V_3-3\,V_4=48 \qquad ......(2)$$

$$V_3=-12\ \text{V}$$

$$V_4=-24\ \text{V}$$

$$"i_1=\frac{-V_3}{6}=\frac{12}{6}=2\ \text{A}$$

$$"i_2=\frac{V_3}{3}=\frac{-12}{3}=-4\ \text{A}$$

$$"i_3=\frac{V_3-V_4}{2}=\frac{-12+24}{2}=6\ \text{A}$$

$$"i_4=\frac{V_4}{4}=\frac{-24}{4}=-6\ \text{A}$$

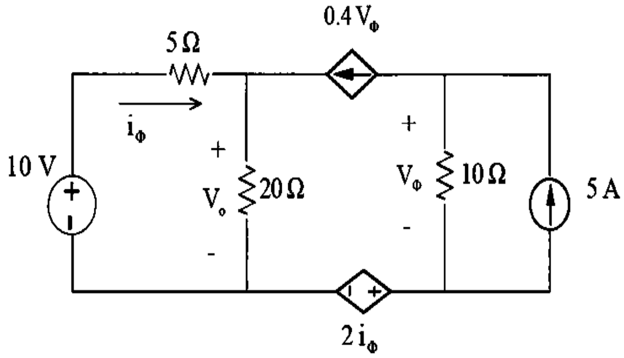
$$i_1='i_1+"i_1=15+2=17\ \text{A}$$

$$i_2='i_2+"i_2=10-4=6\ \text{A}$$

$$i_3='i_3+"i_3=5+6=11\ \text{A}$$

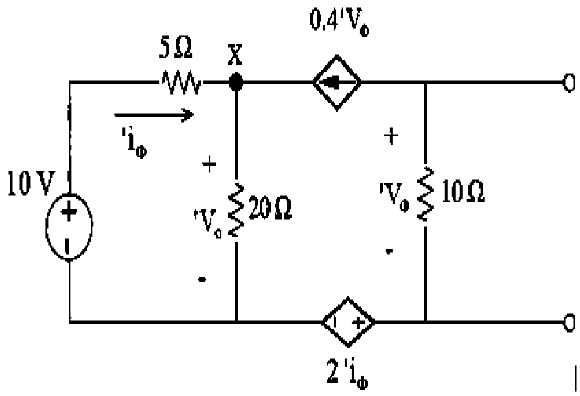
$$i_4='i_4+"i_4=5-6=\text{-}1\ \text{A}$$

2. Use super position to find V0?



Solution:

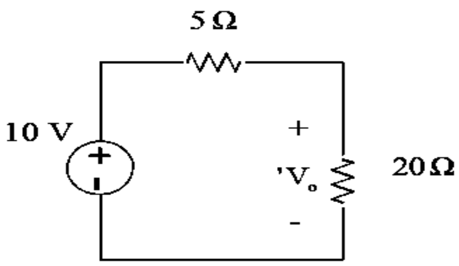
- Activate voltage source only:



$$'V_{\Phi} = 10\,(0.4\,'V_{\Phi}) = 4\,'V_{\Phi}$$

$$'V_{\Phi} = 4\,'V_{\Phi} \quad \Rightarrow \quad 'V_{\Phi} = 0$$

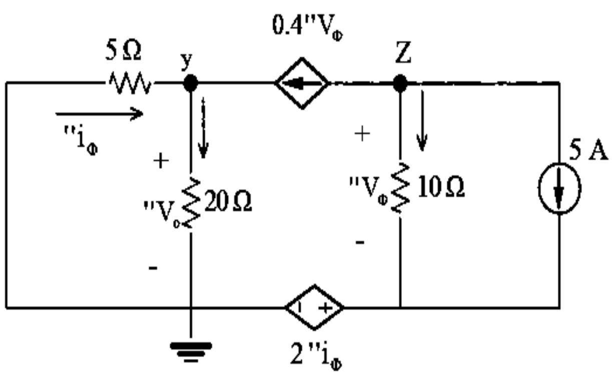
This indicates Dependent current source is open



$$'V_0 = 10\left(\frac{20}{25}\right)$$

$$'V_0 = 8\,V$$

- Activate independent current source only:



KCL at node (y):

$$\frac{-"V_0}{5} - \frac{"V_0}{20} + 0.4\,"V_{\Phi} = 0$$

$$-4\,"V_0 - "V_0 + 8\,"V_{\Phi} = 0$$

$$-5\,"V_0 + 8\,"V_{\Phi} = 0 \qquad ......(1)$$

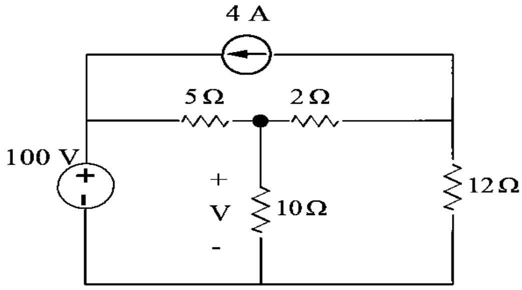
$$5+\frac{''V_{\Phi}}{10}+0.4\,''V_{\Phi}=0$$

$$0.5\,''V_{\Phi}=-5 \quad\Rightarrow\quad ''V_{\Phi}=-10\text{ V}$$

$$\therefore\quad ''V_{0}=\frac{8}{5}\,''V_{\Phi}=\frac{-80}{5}=-16\text{ V}$$

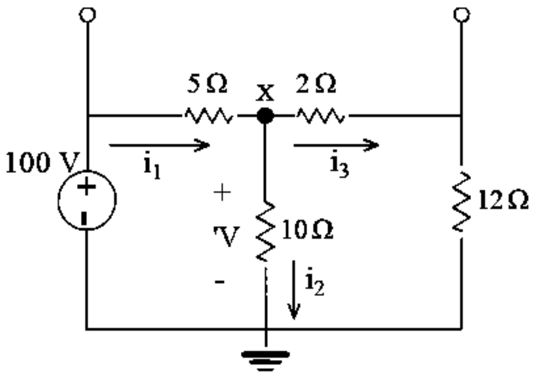
$$V_{0}='V_{0}+''V_{0}=8-16=-8\text{ V}$$

3. Use superposition to find V?



Solution

- Activate voltage source only:



Apply KCL at node (x) :

$$i_1-i_2-i_3=0$$

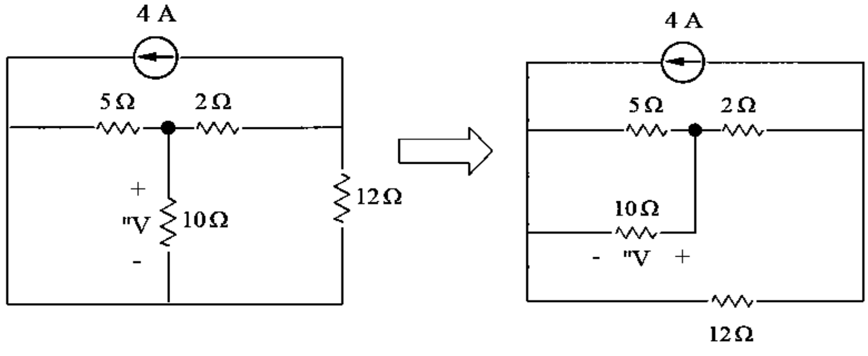
$$\frac{100-'V}{5}-\frac{'V}{10}-\frac{'V}{14}=0$$

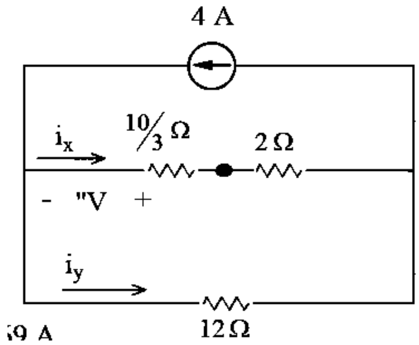
$$22-\frac{'V}{5}-\frac{'V}{10}-\frac{'V}{14}=0$$

$$'V\left[\frac{1}{5}+\frac{1}{10}+\frac{1}{14}\right]=22$$

$$'V=59.23\text{ V}$$

- Activate independent current source only:



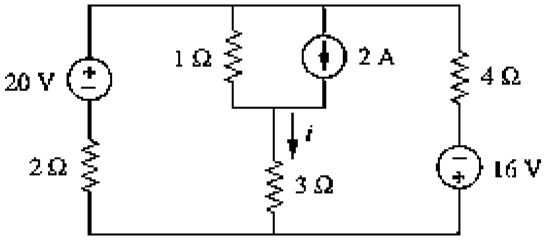


$$i_x = 4\,A\left(\frac{12}{12+\frac{10}{3}+2}\right)=2.769\,A$$

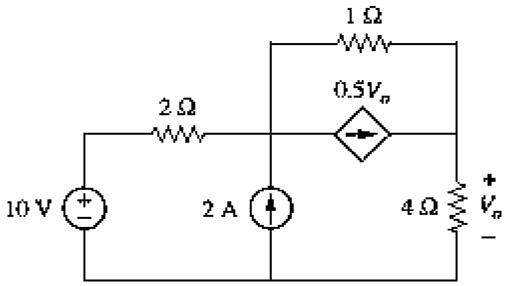
$$''V=-i_x\left(\frac{10}{3}\right)=-9.23\,V$$

$$V='V+''V=59.23-9.23=50\,V$$

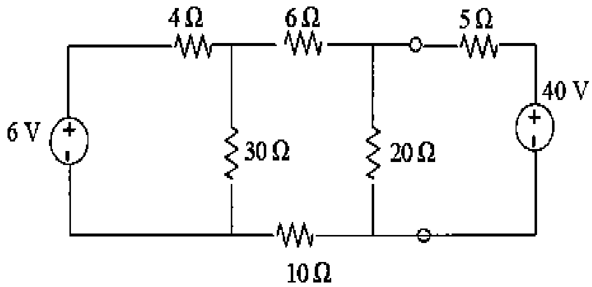
4. For the following circuit, use superposition to find *i*. Calculate the power delivered to the resistor. 3 ohms.



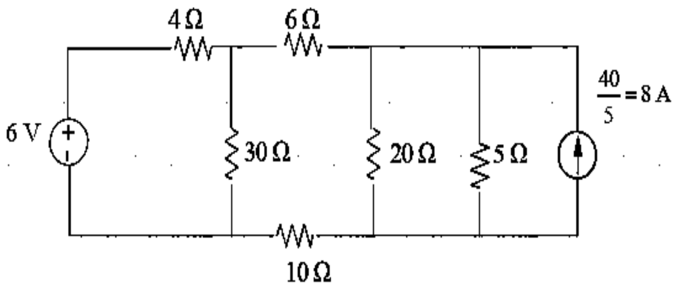
5. Use superposition to find Vo in the circuit



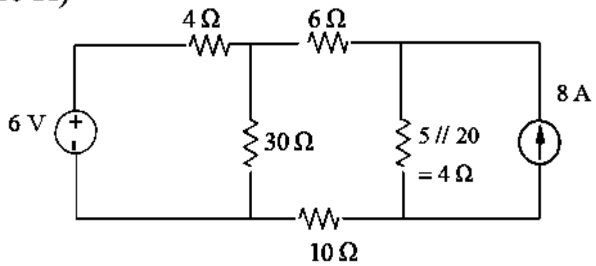
6. Using source transformation, find the power associated with the 6 V source.



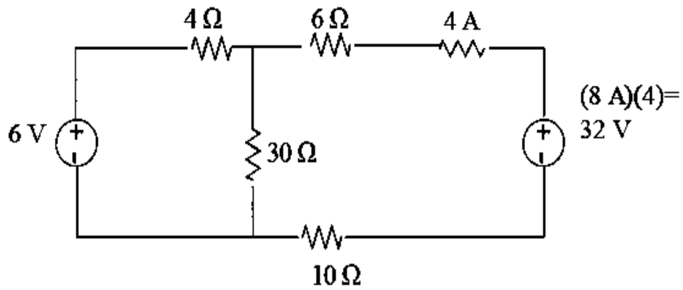
Consider the 40 V source in series with (5Ω)



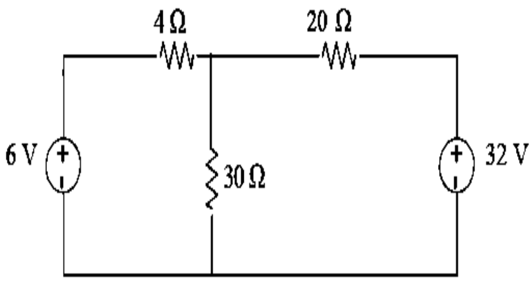
Take (5// 20 Ω)



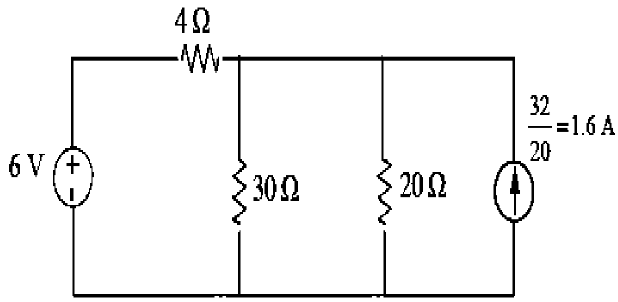
Consider 8A in parallel with (4Ω)



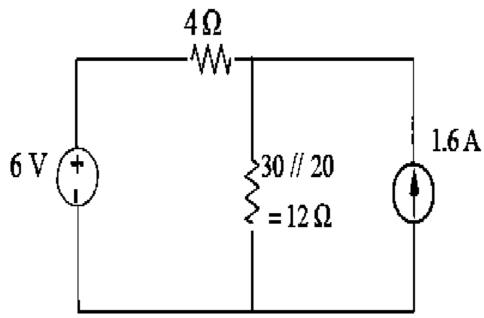
Take (4+6+10) in series



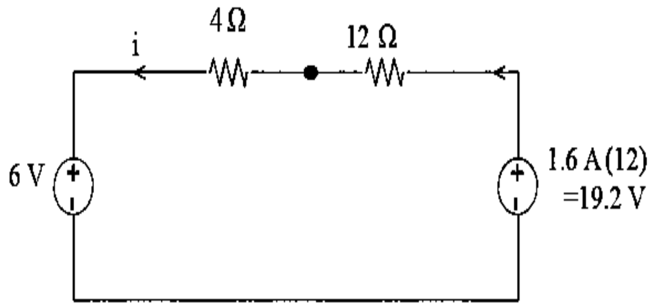
Consider 32 V in series with (20Ω)



Take (30//20 Ω)



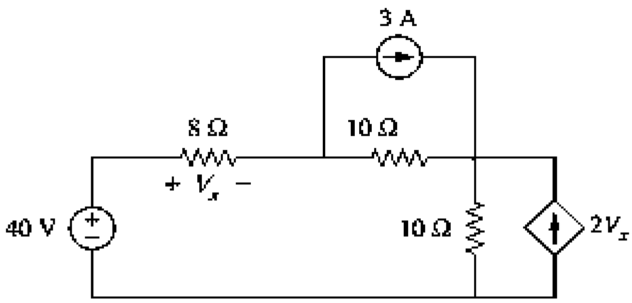
Consider 1.6A in parallel with (15 Ω)



$$i = \frac{19.2 - 6}{4 + 12} = 0.825 \text{ A} \quad \Rightarrow \quad P_{6V} = v\,i = 6\ (0.825)$$

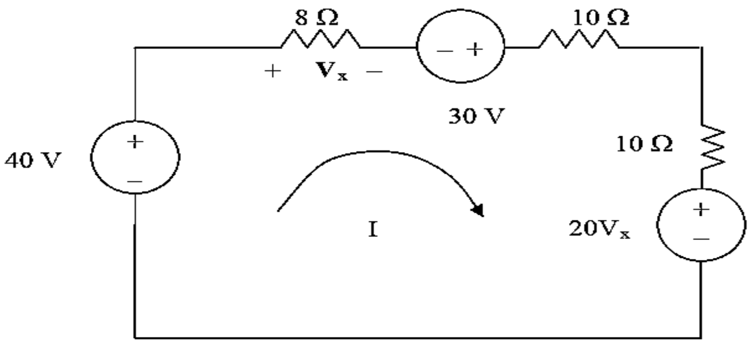
$$P_{6v} = 4.95 \text{ W (absorbing)}$$

7. Use source transformation to find the voltage in Vx the circuit



Solution:

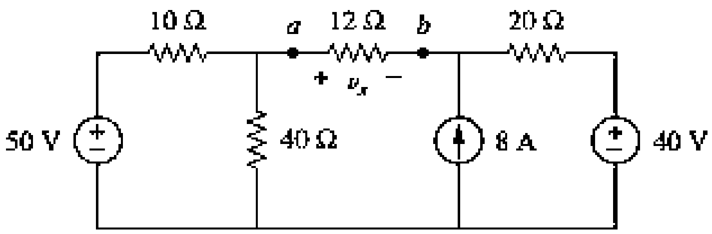
Transform the two current sources in parallel with the resistors into their voltage source equivalents yield, a 30-V source in series with a 10-Ω resistor and a $20V_x$-V sources in series with a 10-Ω resistor.



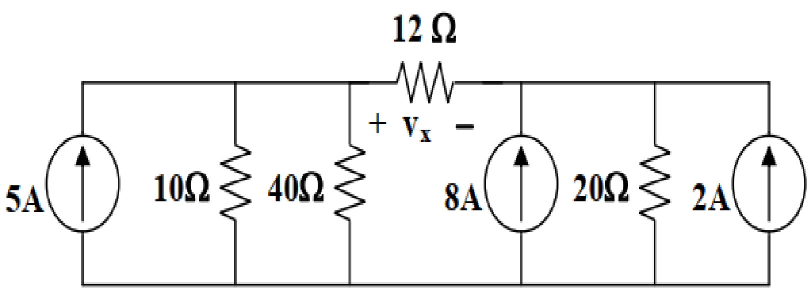
We now write the following mesh equation and constraint equation which will lead to a solution for Vx,

$$28I - 70 + 20Vx = 0 \text{ or } 28I + 20Vx = 70, \text{ but } Vx = 8I \text{ which leads to}$$
$$28I + 160I = 70 \text{ or } I = 0.3723 \text{ A or } Vx = \mathbf{2.978\ V}$$

8. Apply source transformation to find Vx in the circuit

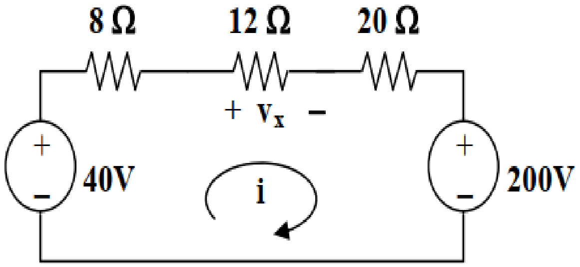


Transforming the voltage sources to current sources gives the circuit



10||40 = 8 ohms

Transforming the current sources to voltage sources yields the circuit

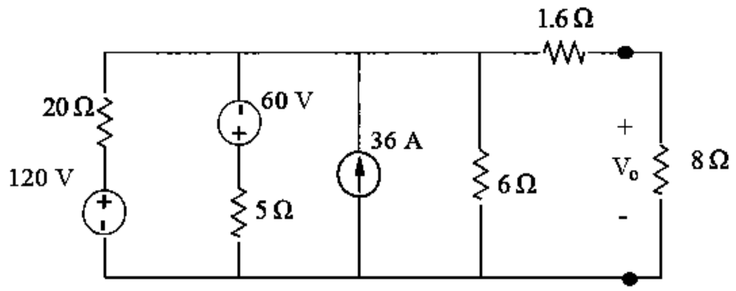


Applying KVL to the loop

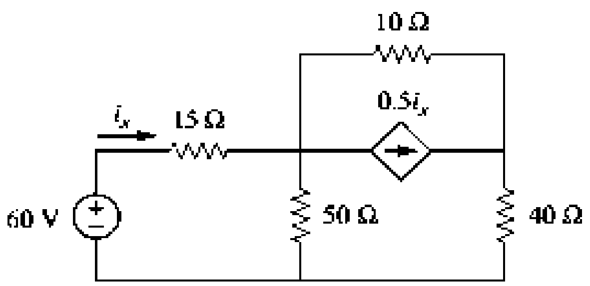
$$-40 + (8 + 12 + 20)i + 200 = 0 \text{ leads to } i = -4$$

$$vx = 12i = \mathbf{-48\ V}$$

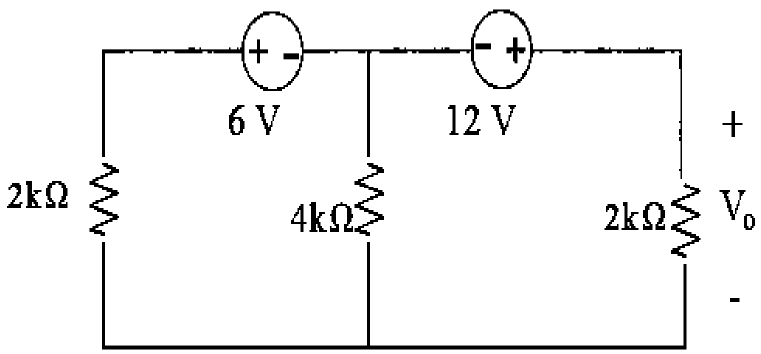
9. Use source transformation to find V0



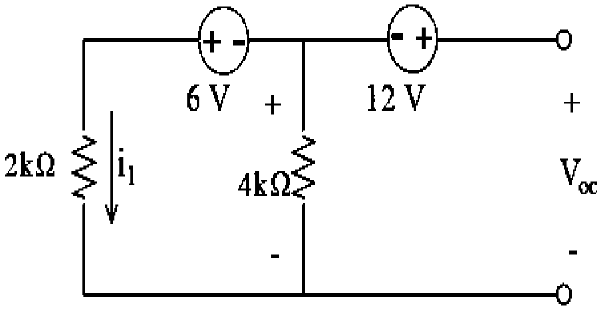
10. Use source transformation to find ix in the circuit



11. Use Thevenin's and Norton Theorms to find V0.



i. Using Thevenin Theorm:
First find VOC and Voc is equal to Thevenin voltage

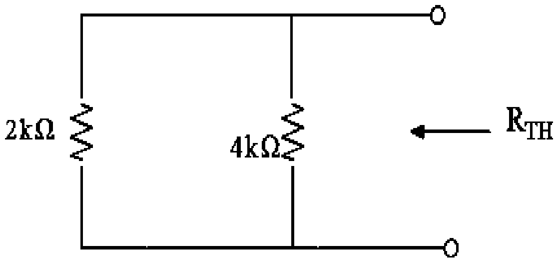


$$i_1 = \frac{6\text{ V}}{2\text{ k}+4\text{ k}} = 1\text{ mA}$$

$$V_{4k\Omega} = i_1(4\text{k}) = -4\text{ V}$$

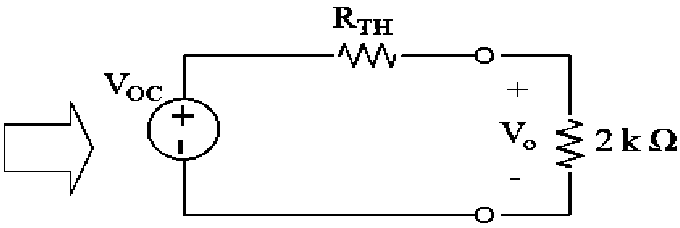
$$V_{oc} = 12\text{ V} - 4\text{ V} = 8\text{ V}$$

Second, find RTH, by making all independent source zero



$$R_{TH} = 2k//4k = 4/3\text{ k}\Omega$$

Thevenin equivalent circuit is

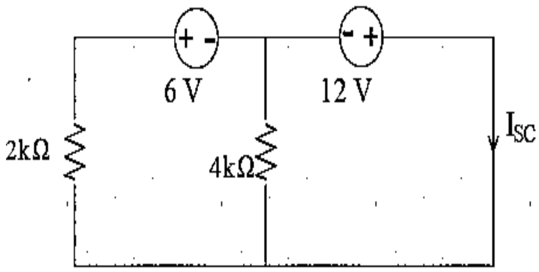


$$V_0 = \frac{2\text{ k}\Omega}{2\text{ k}+R_{TH}} V_{oc}$$

$$= \frac{2\text{ k}}{10/3\text{ k}}(8\text{ V})$$

$$V_0 = 4.8\text{ V}$$

ii. Using Norton Theorm
First find Isc



$$i_1 = \frac{12\text{ V}}{4\text{ k}} = 3\text{ mA}$$

KVL around outer loop:

$$12 - 6 + V_{2k} = 0 \quad \Rightarrow \quad V_{2k} = -6\text{ V}$$

$$i_2 = \frac{V_{2k}}{2\text{ k}} = \frac{-6}{2\text{ k}} = -3\text{ mA}$$

$$i_1 = \frac{12\,V}{4\,k} = 3\,mA$$

KCL at x :

$$i_1 - i_2 - i = 0$$
$$3\,m + 3\,m - i = 0$$

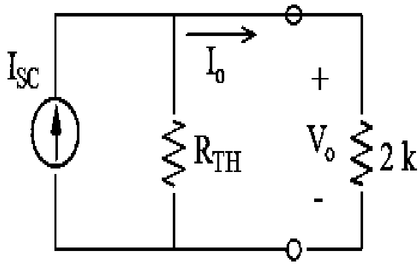
KVL around outer loop:

$$12 - 6 + V_{2k} = 0 \quad \Rightarrow \quad V_{2k} = -6\,V$$

$$i = 6\,mA \quad \Rightarrow I_{sc} = 6\,mA$$

$$i_2 = \frac{V_{2k}}{2\,k} = \frac{-6}{2\,k} = -3\,mA$$

RTH is the same as before:



$$I_0 = \frac{R_{TH}}{R_{TH} + 2\,k}(I_{sc}) = \frac{4/3\,k}{4/3\,k + 2\,k}(6\,m)$$
$$= 2.4\,mA$$
$$V_0 = I_0\,(2\,k) = (2.4\,m)\,(2\,k) = 4.8V$$

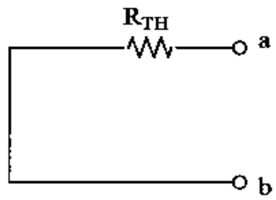
Note : <u>Circuits containing only dependent sources</u>

Here there is <u>NO</u> energy source in the circuit.

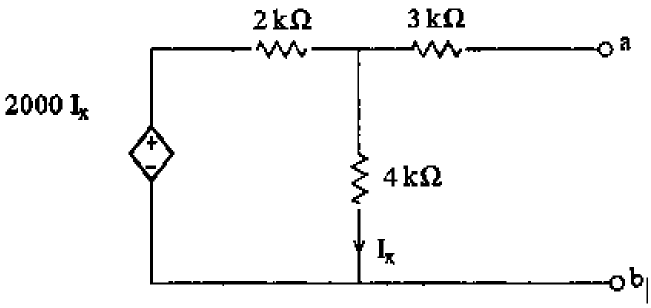
- V_{OC} is always zero and I_{SC} is always zero
- So we can only find R_{TH}

<u>Procedure for finding R_{TH}</u>

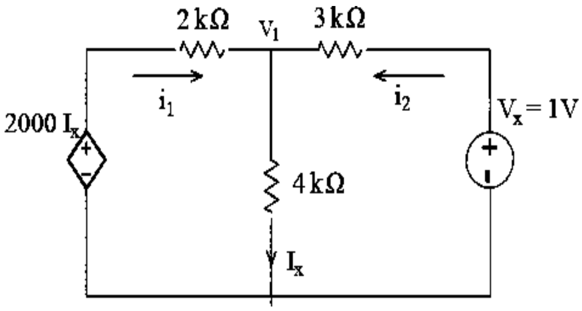
1. Connect an independent voltage (or current) source at the terminals ,Vx (or Ix)
2. Find the corresponding current (or voltage) at the terminal , I_o (or V_o)
3. Find $R_{TH} = Vx / I_o$ or $R_{TH} = V_o / Ix$



12. Find the Thevenin equivalent circuit



Apply voltage source at the terminals (Vx=1V)



KCL at node V1 :

$$i_1 + i_2 - I_X = 0$$

$$\frac{2000\, I_X - V_1}{2\text{ k}} + \frac{1 - V_1}{3\text{ k}} - I_X = 0$$

where $V_1 = (4\text{ k})\, I_X$

$$\frac{2000\, I_X - 4000\, I_X}{2000} + \frac{1 - 4000\, I_X}{3000} - I_X = 0$$

$$I_X - 2\, I_X + \frac{1}{3\text{ k}} - \frac{4}{3\text{ k}} I_X - I_X = 0$$

$$I_X \left[2 + \frac{4}{3} \right] = \frac{1}{3000}$$

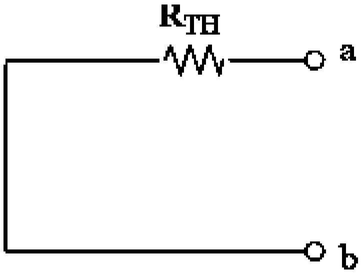
$$I_X = 0.1\text{ m A}$$

$$i_2 = \frac{V_X - V_1}{3\text{ k}}$$

$$= \frac{V_X - (4\text{ k})I_X}{3\text{ k}} = \frac{1 - (4\text{ k})(0.1\text{ m A})}{3\text{ k}}$$

$$i_2 = 0.2\text{ m A}$$

$$R_{TH} = \frac{V_X}{i_2} = \frac{1\text{ V}}{0.2\text{ m A}} = 5\text{ k}\,\Omega$$



Note: Circuits containing both independent and dependent sources

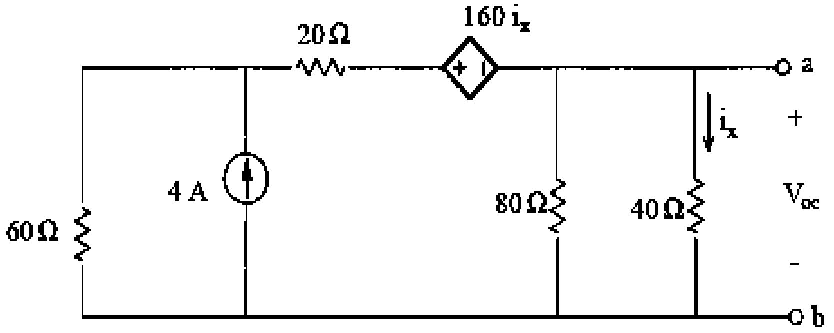
Procedure of Thevenin or Norton Theorms:

a. Find the open circuit voltage and the terminals, VOC

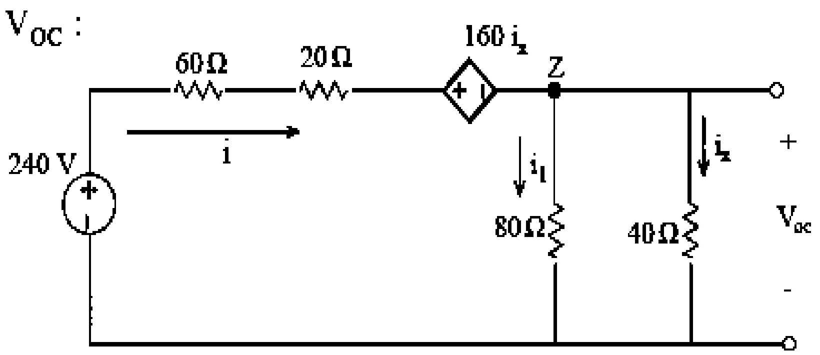
b. Find the short circuit current at the terminals, ISC.

c. Compute RTH = VOC/ISC

13. Find the Thevenin equivalent circuit with respect to the terminals a, b



i. Find VOC



KVL around the lift loop :

$$-240+80\,i+160\,i_x+40i_x=0$$

$$80\,i+200\,i_x=240 \qquad ......(1)$$

KVL around right loop :

$$80\,i_1=40\,i_x$$

$$2\,i_1-i_x=0 \qquad ......(2)$$

KCL at Z:

$$i-i_1-i_x=0 \qquad ......(3)$$

$i=1.125$ A

$i_1=0.375$ A

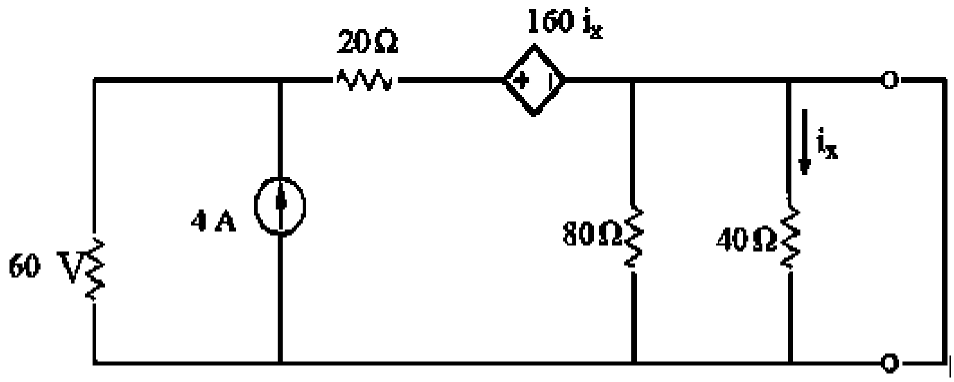
$i_x=0.75$ A

$\therefore \quad V_{OC}=i_x\,(40\ \Omega)$

$= (0.75A)\,(40\ \Omega)$

$V_{OC}=30$ V

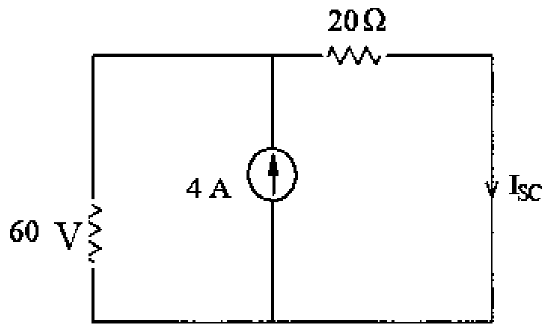
ii Find ISC



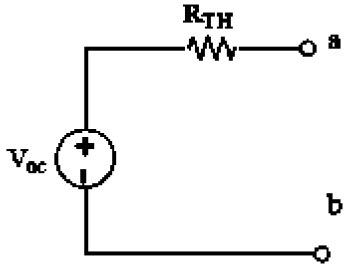
Since we have short circuit, 80 // 40 // 0 = 0

$i_x = 0$

$160 i_x$ source is zero



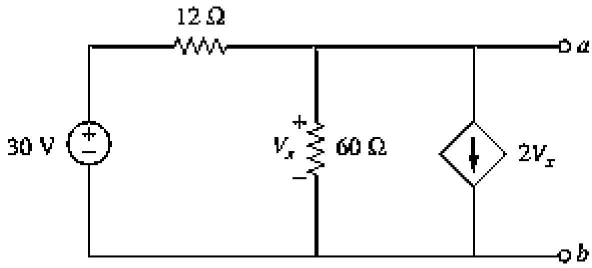
$$I_{SC} = \frac{60}{60+20}(4) = 3\text{ A}$$



Find R_{TH}

$$R_{TH} = \frac{V_{OC}}{I_{SC}} = \frac{30\text{ V}}{3\text{ A}} = 10\ \Omega$$

14. Obtain the Thevenin and Norton equivalent circuits of the circuit shown below with respect to terminals *a* and *b*.

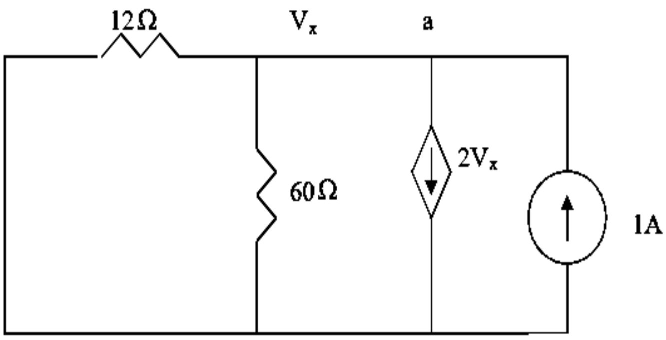


Solution

Since VTh = Vab = Vx, we apply KCL at the node a and obtain

$$\frac{30 - V_{Th}}{12} = \frac{V_{Th}}{60} + 2V_{Th} \quad \longrightarrow \quad V_{Th} = 150/126 = 1.1905\text{ V}$$

To find RTh, consider the circuit below.



At node a, KCL gives

$$1 = 2V_x + \frac{V_x}{60} + \frac{V_x}{12}$$

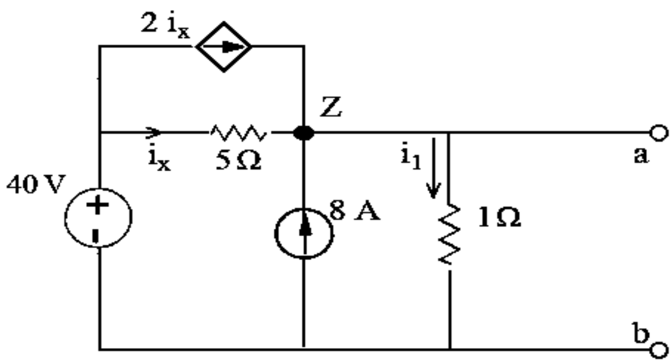
$$V_x = 60/126 = 0.4762$$

$$R_{Th} = \frac{V_x}{1} = 0.4762\Omega, \qquad I_N = \frac{V_{Th}}{R_{Th}} = 1.19/0.4762 = 2.5$$

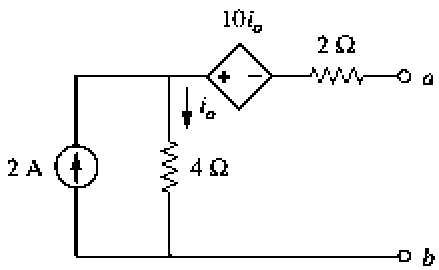
Thus,

VThev = **1.1905 V**, Req = **476.2 mΩ**, and IN = **2.5 A**.

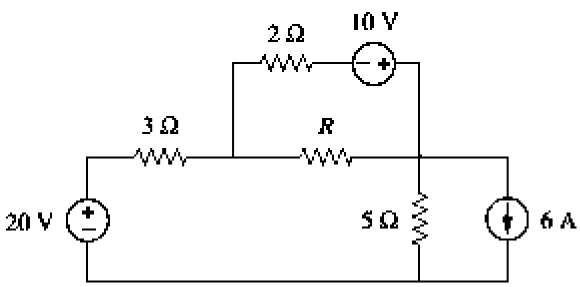
15. Use Thevenin theorem to find the Thevenin equivalent circuit with respect to a, b



16. Determine the Norton equivalent at terminals *a-b* for the circuit shown below

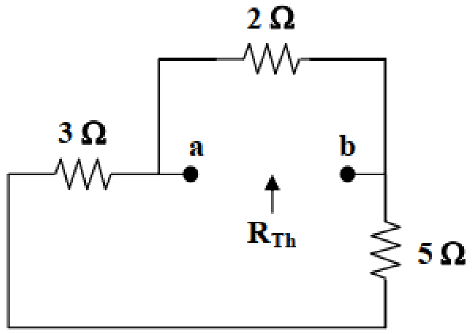


17. Find the maximum power that can be delivered to the resistor *R* in the circuit shown below.

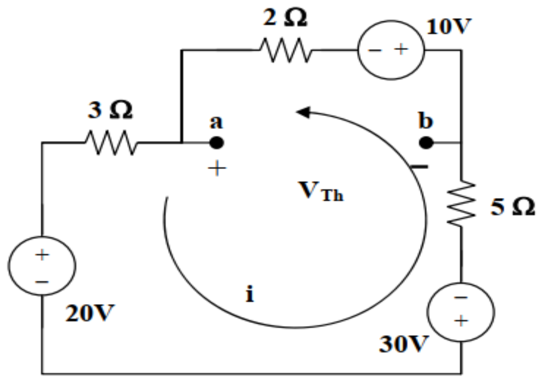


Solution

We first find the Thevenin equivalent at terminals a and b. We find RTh using the circuit (a)



(a)

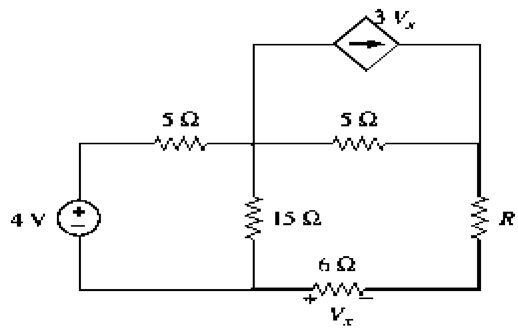


(b)

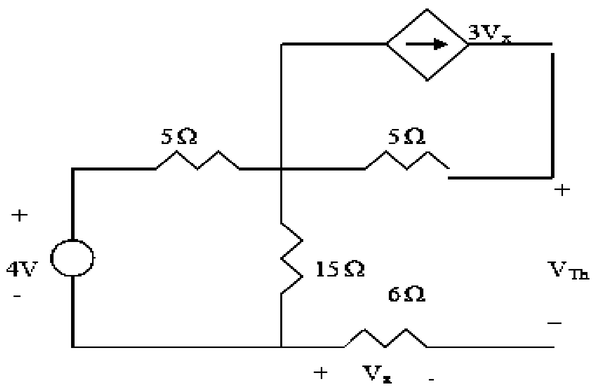
By performing source transformation on the given circuit, we obatin the circuit in (b). We now use this to find V_{Th}.

$$10i + 30 + 20 + 10 = 0, \text{ or } i = -6$$
$$V_{Th} + 10 + 2i = 0, \text{ or } V_{Th} = 2 \text{ V}$$
$$p = V_{Th}^2/(4R_{Th}) = (2)^2/[4(1.6)] = \mathbf{625\ m\ watts}$$

18. Determine the maximum power delivered to the variable resistor *R* shown in the circuit of shown below



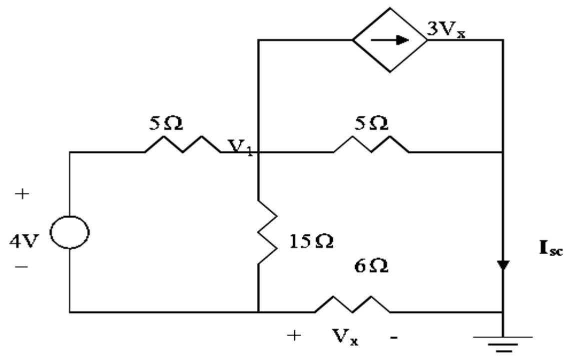
Solution: We find the Thevenin equivalent across the 10-ohm resistor. To find V_{Th}, consider the circuit below.



From the figure,

$$V_x = 0, \qquad V_{Th} = \frac{15}{15+5}(4) = 3\text{V}$$

To find Req, consider the circuit below:



At node 1,

[(V1–Vx)/15] + [(V1–(4+Vx))/5] + [(V1–0)/5] + 3Vx = 0 or

0.4667V1 + 2.733Vx = 0.8 (1)

At node x,

[(Vx–0)/6] + [((Vx+4)–V1)/5] + [(Vx–V1)/15] = 0 or

–(0.2667)V1 + 0.4333Vx = –0.8 (2)

Adding (1) and (2) together lead to,

(0.4667–0.2667)V1 + (2.733+0.4333)Vx = 0 or V1 = –(3.166/0.2)Vx = –15.83Vx

Now we can put this into (1) and we get,

0.4667(–15.83Vx) + 2.733Vx = 0.8 = (–7.388+2.733)Vx = –4.655Vx or

Vx = –0.17186V

Isc= –Vx/6 = 0.02864 and Req = 3/(0.02864) = 104.75 Ω

An alternate way to find Req is replace Isc with a 1-amp current source flowing up and setting the 4 volts source to zero. We then find the voltage across the 1-amp current source which is equal to Req. First, we note that Vx = 6 volts;

V1 = 6+3.75 = –9.75; V2 = 19x5 + V1 = 95+9.75 = 104.75 or Req = 104.75 Ω

Clearly setting the load resistance to 104.75 Ω means that the circuit will deliver maximum power to it. Therefore

pmax = [3/(2x104.75)]2x104.75 = **21.48 mW**

19. Compute the value of *R* that results in maximum power transfer to the 10- resistor and Find the maximum power.